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A Monotone Process Repair Model For A Repairable Cold Standby System With Priority In Use And Repair

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Abstract—The purpose of this paper is to apply the two monotone process repair model to a two-dissimilar – component cold standby repairable system with one repairman and priority in use and repair. Assume that either component after repair is not "as good as new" and follows a monotone process repair, and component 1 has priority in use and repair under these assumptions, we consider a replacement policy N based on the number of repairs of component 1 under which the system is replaced when the repair number of component 1 reaches N. The problem is to determine an optimal replacement policy N* such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically.

Key words—Renewal reward theorem, Average long run cost rate, Replacement policy, Geometric process, Alpha series process.

I. INTRODUCTION

In modern study, the maintenance problems are concern with simple repairable system under is "as good as new". However, this assumption is not always true in practical applications. Barlow and Hunter [1] developed a minimal repair model in which the minimal repair does not change the age of the system. Brown and Proschan [2] studied an imperfect repair model under which the repair will be perfect repair with probability'p' and with probability '(1-p)' as a minimal repair. It is reasonable to assume that the successive working times of the deteriorating systems after repair will become shorter and shorter, while the consecutive repair time of the system will become longer and longer. Finally, neither it works nor repaired any more.

Stadje and Zukerman [8]introduced a general monotone process repair model to generalize Lam's work. Zhang [9] combined the two replacement policies used by Lam [4,5] and proposed a bivariate replacement policy (T,N) under which the system is replacement at the working age T or at the time of the Nth failure, whichever occurs first and showed that the optimal policy (T,N)* is better than the optimal policy N* and T*.

In general, stand by techniques are used to improve system reliability, raise system availability or reduce system cost Zhang[10] introduced a two- identical – component cold standby repairable system with one repairman.

By using the geometric process repair model he studied a replacement policy N based on the number of repairs of component. An optimal replacement N* by maximizing the long-run expected reward per unit time is determined. Likewise, priority rules in the multi-component repairable systems are also used to improve system reliability, raise system availability or reduce system cost. Thus, a standby redundant system consisting of a main component with priority in use or repair and a standby component is often installed in practical situations. For example, in the operating room of a hospital, an operation must be discontinued as soon as the power source is cut (i.e. power station failures). Usually, there is a standby power station (e.g. a storage battery) in the operating room. Thus, the power station (regarded as the main component, e.g. component 1) and the storage battery (regarded as the standby component, e.g. component 2) form a cold standby repairable lighting system. Obviously, it is reasonable to assume that the power station has use priority due to the operating cost of the power station is cheaper than the operating cost of the storage battery, and power station has repair priority due to the used area of power station is wider than the storage battery (only in the operating room) Besides a cold standby lighting system in a hospital, some similar examples can be found from Lam[6].

The purpose of this paper is to apply the two monotone process repair model to a two-dissimilar – component cold standby repairable system with one repairman and priority in use and repair. Assume that either component after repair is not "as good as new" and follows a monotone process repair, and component 1 has priority in use and repair under these assumptions, we consider a replacement policy N based on the number of repairs of component 1 under which the system is replaced when the repair number of component 1 reaches N. The problem is to determine an optimal replacement policy N* such that the average cost rate of the system is minimized. The explicit equation of the average cost rate of the system is derived and the corresponding optimal replacement policy N* can be determined analytically or numerically.

In modeling these deteriorating systems, the definitions according to Lam [4,5] are considered.

II. THE MODEL

We study a two-component cold standby repairable system with one repairman and priority in use and repair by making the following assumptions:

1. At the beginning two components are both new, and component 1 is in a working state while component 2 is in a cold standby state.
2. Assume that both the components after repair are not as 'good as new' and follow a two monotone process repair.
3. Assume that when components are good, component has the priority in use than the component 2, even if component 2 is working, it must be switched into the cold standby state as soon as components after failure has been repaired, and it becomes the working state immediately.
4. When both components fail the system is down), component has the higher repair priority than component 2. Even of the repairmen is operating component 2 at this stage, he must switch to component 1. He will work on the repair of component 2 after completing the repair on component 1. A possible course of the system is shown in Fig 1.
5. Assume that the time interval between the completion of the (n-1)th repair and the completion of the nth repair of component 'i' is called the nth cycle of component i, i21,2, n=1,2,3……..
6. Assume that $X_n^{(i)}$ and $Y_n^{(i)}$ are working time and repair time of component 'i' respectively and the consecutive working time follow a decreasingly α - series process while the consecutive repair times follow and increasing geometric process.
7. Assume that the replacement policy N based on the number of repairs of component 1is used when the system is down, the system will be replaced by a new and identical one is negligible and the replacement time.
8. Assume that any component in the system can't produce the working reward during cold standby and no cost is incurred during waiting for repair.
9. Assume that the repair cost rate of component 'i' is $C_n^{(i)}$, i =1,2 while the working reward rate of two component is same C_W. And the replacement cost of system is C.

III. THE LONG RUN AVERAGE COST PER UNIT OF TIME

In this section, we consider two replacement policy N based on the number of repairs of component 1. Because the two components appear alternately in the system. When the repair number of component, reaches N, then component, should work until failure in the $(n+1)^{th}$ cycle whatever component 2 is either in the working state or waiting for repair state in the Nth cycle. Thus the renewal point under the policy N is found.

Let T be first replacement time of the system and T_m et (n>2) be the time between the (n-1) the replacement and the nth replacement of the system under policy N. Obviously, $\{T_1, T_{2\ldots\ldots}\}$ form a renewal process and the interval time between two consecutive replacements is called a renewal cycle.

Let C (N) be the average cost rate of the system under policy N, thus, according to renewal reward theorem Ross [7] we have:

$$C(N) = \frac{The\ \exp ected\ \ \ \cos t\ incurred\ \ in\ a\ renewal\ cycle}{The\ \exp ected\ \ \ length\ of\ \ a\ renewal\ cycle} \qquad (3.1)$$

Let w be the length of a renewal cycle of the system under the policy N. Because component 1 has priority in use and repair, component 1 only resides in the working state and the repair state. Thus, based on the enactment of the renewal point under policy N, we have

$$W = \sum_{j=1}^{N+1} X_j^{(1)} + \sum_{j=1}^{N} Y_j^{(1)} \qquad (3.2)$$

Where the first term and the second term are, respectively, the length of working time and the length of repair time of component, before the number of repairs of component reaches N.

The total working time 'L' and the total repair time 'R' of the system in a renewal cycle are respectively given by:

$$L = \sum_{j=1}^{N+1} X_j^{(1)} + \sum_{j=1}^{N} X_j^{(2)} I_{\left\{Y_{j-1}^{(2)} - X_j^{(1)} < 0\right\}} \qquad (3.3)$$

$$R = R_1 + R_2 = \sum_{j=1}^{N} Y_j^{(1)} + \sum_{j=1}^{N} Y_j^{(2)} I_{\left\{X_{j-1}^{(2)} - Y_j^{(1)} < 0\right\}} \qquad (3.4)$$

Now, the expectation of W, L and R are respectively given by:

$$E(W) = \sum_{j=1}^{N+1} E\left(X_j^{(1)}\right) + \sum_{j=1}^{N} E\left(Y_j^{(1)}\right), \qquad (3.5)$$

$$E(L) = \sum_{j=1}^{N+1} E\left(X_j^{(1)}\right) + \sum_{j=1}^{N} E\left(X_j^{(2)}\right) E\left(I_{\left\{Y_{j-1}^{(2)} - X_j^{(1)} < 0\right\}}\right), \qquad (3.6)$$

$$E(R) = E(R_1) + E(R_2) = \sum_{j=1}^{N} E\left(Y_j^{(1)}\right) + \sum_{j=1}^{N} E\left(Y_j^{(2)}\right) E\left(I_{\left\{X_{j-1}^{(2)} - Y_j^{(1)} < 0\right\}}\right). \qquad (3.7)$$

Where R_1 and R_2 denote, respectively, the repair time of the component 1 and 2 in a renewal cycle, I is the indicator function such that

$$I_A \begin{cases} = 1 \ \ if\ event\ A\ occurs \\ = 0 \ \ if\ event\ A\ doesn't\ occurs \end{cases}$$

According to assumptions of the model and the definition of the convolution, let the distribution functions of $\left(Y_{k-1}^{(2)} - X_k^{(1)}\right)$ *and* $\left(X_k^{(2)} - Y_k^{(1)}\right)$ are respectively $\boldsymbol{\Phi}_k(u)$ *and* $\boldsymbol{\Psi}_k(v)$.

Where

$$\Phi_k(u) = G^{(2)}(b_2^{k-2}u) * \left[1 - F^{(1)}(-a_1^{k-1}u)\right] \quad (3.8)$$

$$\Psi_k(v) = F^{(2)}(a_2^{k-1}v) * \left[1 - G^{(1)}(-b_1^{k-1}v)\right], \quad (3.9)$$

and * denotes convolution.
using the definition of the distribution functions and the concept of the condition distribution, we have

$$\Phi_k(0) = P\left(Y_{k-1}^{(2)} - X_j^{(1)} < 0\right) = \int_0^\infty G_{k-1}^{(2)}(t)\, d F_k^{(1)}(t) \quad (3.10)$$

$$\Psi_k(0) = P\left(X_k^{(2)} - Y_k^{(1)} < 0\right) = \int_0^\infty F_k^{(2)}(t)\, d G_k^{(1)}(t) \quad (3.11)$$

Since, it is assumed that $X_k^{(i)}$ *and* $Y_k^{(i)}$, *for* $i = 1,2.$ are all exponential, then their distribution functions are given by

$$F_k^{(i)}(x) = F^{(i)}(k^{\alpha_i} x) = 1 - \exp(-k^{\alpha_i}\lambda_i x), \text{ for } i = 1,2.$$

$$G_k^{(i)}(y) = G^{(i)}(b_i^{k-1} y) = 1 - \exp(-b_i^{k-1}\mu_i y), \text{ for } i = 1,2. \ ,$$

$$\text{where } x \geq 0,\ y \geq 0,\ 0 \leq \alpha_i \leq 1,\ \ 0 \leq b_i \leq 1,$$

$$E\left(X_k^{(i)}\right) = \int_0^\infty x dF_k^{(i)}(k^{\alpha} x) = \frac{1}{\lambda_i k^{\alpha_i}},\ i = 1,2. \quad (3.12)$$

$$E\left(Y_k^{(i)}\right) = \int_0^\infty y dG_k^{(i)}(b^{k-1} y) = \frac{1}{\mu_i b_i^{k-1}},\ i = 1,2. \quad (3.13)$$

According to the assumptions of the model, definition of probability density function, convolution and Jacobian transformations , the probability density functions of $u = (Y_{j-1}^{(2)} - X_j^{(1)})$ and $v = (X_j^{(2)} - Y_j^{(1)})$ are respectively, $\phi_j(u)$ and $\psi_j(v)$.

$$\phi_j(u) = \int_0^\infty f(v, u+v)\,dv, \quad (3.14)$$

Where $X_k^{(1)} = v,\ Y_{k-1}^{(2)} = u + v,$ such that $u = Y_{k-1}^{(2)} - X_k^{(1)}$,

$$\phi_k(u) = \int_0^\infty f(v, u+v)\,dv = \begin{cases} \dfrac{k^{\alpha_1} b_2^{k-2}\lambda_1\mu_2}{k^{\alpha_1}\lambda_1 + b_2^{k-2}\mu_2} e^{-b_2^{k-2}\mu_2 u} & \text{for } u \geq 0 \\ \dfrac{k^{\alpha_1} b_2^{k-2}\lambda_1\mu_2}{k^{\alpha_1}\lambda_1 + b_2^{k-2}\mu_2} e^{-k^{\alpha_1}\lambda_1 u} & \text{for } u < 0 \end{cases}$$

(3.15)

Let $\psi(v) = \int_0^\infty f(u+v, u)\,du$ (3.16)

where $X_k^{(2)} = u + v;\ Y_k^{(1)} = u$ *such that* $v = X_k^{(2)} - Y_k^{(1)}$.

$$\psi_k(v) = \int_0^\infty f(u+v, u)\,du$$

$$\psi_k(v) = \int_0^\infty f(u+v,u)\,du = \begin{cases} \dfrac{k^{\alpha_2} b_1^{k-1} \lambda_2 \mu_1}{k^{\alpha_2}\lambda_2 + b_1^{k-1}\mu_1} e^{-k^{\alpha_2}\lambda_2 v} & for\ v \geq 0 \\ \dfrac{k^{\alpha_2} b_1^{k-1} \lambda_2 \mu_1}{k^{\alpha_2}\lambda_2 + b_1^{k-1}\mu_1} e b_1^{k-1}\mu_1 v & for\ v < 0 \end{cases} \tag{3.17}$$

$$E[(Y_{k-1}^{(2)} - X_k^{(1)}) I_{(Y_{k-1}^{(2)} - X_k^{(1)})>0}] = \int_0^\infty u\,\phi_k(u)\,du\,, \tag{3.18}$$

Using equation (3.15), we have:

$$= \frac{k^{\alpha_1}\lambda_1}{(k^{\alpha_1}\lambda_1 + b_2^{k-2}\mu_2) b_2^{k-2}\mu_2}, \quad k \geq 2 \tag{3.19}$$

Let $$E[(X_{k-1}^{(2)} - Y_k^{(1)}) I_{(X_{k-1}^{(2)} - Y_k^{(1)})>0}] = \int_0^\infty v\,\psi_k(v)\,dv \tag{3.20}$$

Using equation (3.17), we have:

$$= \frac{b_1^{k-1}\mu_1}{(k^{\alpha_2}\lambda_2 + b_1^{k-1}\mu_1) k^{\alpha_2}\lambda_2}\quad , k \geq 1 \tag{3.21}$$

Using equation (3.10), we have:

$$\Phi_k(0) = p(Y_{k-1}^{(2)} - X_k^{(1)} < 0) = \frac{b_2^{k-2}\mu_2}{(k^{\alpha_2}\lambda_2 + b_2^{k-2}\mu_2)}, \quad k \geq 2 \tag{3.22}$$

Using equation (3.11), we have :

$$\psi_k(0) = p(X_k^{(2)} - Y_k^{(1)} < 0) = \frac{k^{\alpha_2}\lambda_2}{(k^{\alpha_2}\lambda_2 + b_1^{k-1}\mu_1)}, \quad k \geq 1 \tag{3.23}$$

Using equation (3.23), we have:

$$E\left[X_k^{(2)} I_{(Y_{k-1}^{(2)} - X_k^{(1)} < 0)}\right] = \frac{1}{\lambda_2 a_2^{k-1}} \Phi_k(0)$$

$$= \frac{1}{\lambda_2 k^{\alpha_2}} \frac{b_2^{k-2}\mu_2}{(k^{\alpha_2}\lambda_2 + b_2^{k-2}\mu_2)} \tag{3.24}$$

Using equation (3.23), we have:

$$E\left[Y_k^{(2)} I_{(X_k^{(2)} - Y_k^{(1)} < 0)}\right] = \frac{1}{\mu_2 b_2^{k-1}} \Psi_k(0)$$

$$= \frac{1}{\mu_2 b_2^{k-1}} \frac{k^{\alpha_2}\lambda_2}{(k^{\alpha_2}\lambda_2 + b_1^{k-1}\mu_1)} \tag{3.25}$$

By using equation (3.12) and (3.13), equation (3.5) becomes:

$$E(W)=\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{j=1}^{N}\frac{1}{b_1^{\ k-1}\mu_1} \qquad (3.26)$$

By using equation (3.12) and (3.24), equation (3.6) becomes:

$$E(L)=\sum_{j=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{b_2^{k-2}\mu_2}{\left(k^{\alpha_2}\lambda_2+b_2^{\ k-2}\mu_2\right)} \qquad (3.27)$$

By using equation (3.13) and (3.25), equation (3.7) becomes:

$$E(R)=\sum_{j=1}^{N}\frac{1}{\mu_1 b_1^{k-1}}+\sum_{k=1}^{N}\frac{1}{\mu_2 b_2^{k-1}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2+b_1^{\ k-1}\mu_1\right)} \qquad (3.28)$$

Substituting equations (3.26) , (3.27) and (3.28) into equation (3.1), then the average cost rate of the system under policy N is given by

$$C(N)=\frac{C_r^{(1)}E(R_1)+C_r^{(2)}E(R_2)+C-C_w E(L)}{E(W)} \qquad (3.29)$$

$$C(N)=\frac{\left[C_r^{(1)}\sum_{j=1}^{N}\frac{1}{\mu_1 b_1^{k-1}}+C_r^{(2)}\sum_{k=1}^{N}\frac{1}{\mu_2 b_2^{k-1}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2+b_1^{\ k-1}\mu_1\right)}+C-C_w\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{b_2^{k-2}\mu_2}{\left(k^{\alpha_2}\lambda_2+b_2^{\ k-2}\mu_2\right)}\right]}{\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}+\sum_{j=1}^{N}\frac{1}{b_1^{\ k-1}\mu_1}} \qquad (3.30)$$

$$C(N)=\frac{C_r^{(1)}l_1+C_r^{(2)}l_3+C-C_w(l_1+l_4)}{l_1+l_2} \qquad (3.31)$$

Where $l_1=\sum_{k=1}^{N+1}\frac{1}{\lambda_1 k^{\alpha_1}}$, $l_2\sum_{j=1}^{N}\frac{1}{b_1^{\ k-1}\mu_1}$, $l_3=\sum_{k=1}^{N}\frac{1}{\mu_2 b_2^{k-1}}\frac{k^{\alpha_2}\lambda_2}{\left(k^{\alpha_2}\lambda_2+b_1^{\ k-1}\mu_1\right)}$, and $l_4=\sum_{k=2}^{N}\frac{1}{\lambda_2 k^{\alpha_2}}\frac{b_2^{k-2}\mu_2}{\left(k^{\alpha_2}\lambda_2+b_2^{\ k-2}\mu_2\right)}$.

IV. NUMERICAL RESULTS AND CONCLUSION

For the given hypothetical values of the parameters λ_1=10, λ_2=5,μ_1=4, μ_2=6,b1=0.95, b2=0.75,C=350,Cw=20.

Table : 4.1 The average cost rate values

	α_1=0.65, α_2=0.85	α_1=0.35, α_2=0.45	α_1=0.25, α_2=0.35	α_1=0.15, α_2=0.25
N	C(N)	C(N)	C(N)	C(N)
2	845.6312125	815.8708553	805.0275132	793.7139672
3	488.3439205	465.0383618	456.3400145	447.1389811
4	346.1264336	326.8255987	319.4678499	311.5885245
5	270.1859856	253.5193915	247.05234	240.0521656
6	223.6708772	208.8130662	202.9625177	196.5699941

7	193.0807668	179.478805	174.0565047	168.0825226
8	172.3510239	159.6009095	154.4643959	148.7627813
9	158.4179023	146.2016904	141.2339436	135.681459
10	149.6416992	137.6919157	132.789705	127.2745018
11	145.1520833	133.2296024	128.2960203	122.710088
12	**144.548016**	**132.4270492**	**127.3661825**	**121.5997799**
13	147.7535025	135.2104968	129.9232078	123.8602708
14	154.950258	141.7549579	136.1352119	129.6491813
15	166.5531369	152.4607844	146.3921854	139.3419194
17	183.2128659	167.956897	161.3092093	153.5347555
18	205.8392774	189.1240809	181.7495785	173.0676413
19	235.6426956	217.1361256	208.865646	199.0646177
20	274.1937682	253.5191732	244.1577749	232.9922242
21	323.5038876	300.2314714	289.5536079	276.7381339
22	415.0634944	405.7501299	401.5547947	396.2020074

V. CONCLUSIONS

a) From the table 4.1 , We can examine that the long-run average cost per unit time at the time C (12) = **144.548016** is minimum at α_1=0.65 and α_2=0.85**.** We should replace the system at the time of 12th failure.

b) From the table 4.1,we examined that the long-run average cost per unit time at the time C (12) = **127.36618** is minimum at α_1=0.25 and α_2=0.35**.** We should replace the system at the time of 12th failure. Thus from (a) and (b), it can conclude that the value of 'α_1, α_2 and' the long-run average cost per unit time are positively related while the optimal number of failures are constant.

c) If the repairman experiences with repair then the successive repair times form a decreasing geometric process, while the consecutive working times form a an increasing alpha process. Thus this model can also be applied for an improved model.

d) From the table 4.1 ,we examine the optimal failure number of failures, the long-run average cost per unit time and the parameter of the process as follows:

Table Number	Optimal number of failures	The long-run average cost per unit time	Parameter of the process
4.1	**12**	**144.5480**	α_1=0.65 andα_2=0.85
4.1	**12**	**132.42704**	α_1=0.35 and α_2=0.45
4.1	**12**	**127.3661**	α_1=0.25 and α_2=0.35
4.1	**12**	**121.5997**	α_1=0.15 and α_2=0.25

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