Performance Evaluation of Downlink LTE Cellular Network with a Volterra based filter Equalizer

Sunil R. Gosavi¹, Mukund G. Wani²

¹,²Electronics and Telecommunication Department
Dr. D.Y.P. SOET, Lohegaon, Pune
¹Email: srgosavi@hotmail.com

Abstract—In this paper a channel estimation algorithm based on Volterra technique for LTE Downlink System is proposed. According to the LTE standard, coherent detection is used and thus to improve a high data rate over mobile radio channels. For estimation purpose the LTE standard provides known data as pilots symbols. Our aim is to best exploit reference signal to improve system performance by using powerful estimation technique like volterra. Principally the full volterra equalizer is designed to the nonlinear system. In this work, we adapt this powerful technology to the Downlink transmission in LTE System to minimize interferences and nonlinearity effects introduced by multipath transmission and OFDM Transmitter. In this paper, we present the comportment of Volterra equalizer in multipath LTE System in term of BLER, EVM (%) versus SNR and in term of received constellation diagram. Our results are compared with basic LMS equalizer.

Keywords—LTE, Volterra, LMS, Orthogonal Frequency Division Multiple Access (OFDMA), BLER

I. INTRODUCTION

In the modern world, the requirement of high data rate communication has become inevitable. The applications like transmission of Streaming, video and images, and browsing the World Wide Web require high speed data transmission with mobility. The LTE system provides an important effective bit rate and allows increasing system capacity in terms of numbers of simultaneous calls per cell. In addition, it has a low latency compared to 3G/3G+ networks. It offers a speed of 100 Mbits / s in the downlink and 50 Mbits/s in the uplink direction. The LTE uses OFDM modulation and multiple access technique OFDMA in the downlink connection [1]. The OFDM provides to the transmitted signal robustness against the multipath effect and can improve the spectral efficiency of the system [2,3]. On the other hand, the implementation of a MIMO system increases channel capacity and decreases the signal fading by sending the same information at the same time through multiple antennas [3]. The combination of these two powerful technologies (MIMO-OFDM) in the LTE system improving thus the spectral efficiency and throughput offered without increasing resources for base bands and power output. In addition, Channel estimation plays a crucial role for improving the performance of LTE Downlink system. Many investigations have been conducted in this way.

Actually the trend is to use iterative equalizer because of their great performances [4,5,6]. For this work, we exploit the known pilots provided by LTE standard by using powerful technique of estimation like the full Volterra algorithm. In fact, we adapt the Volterra algorithm for LTE system to minimize interference and nonlinearity effects introduced respectively by multipath transmission and OFDM transmitter. We compare performance of our algorithm with one of most popular equalization LMS in term BLER, EVM (%) versus SNR and in term of received constellation diagram. The remainder of the paper is organized as follows., in section II we give an overview about physical layer of LTE System, we present also the MIMO-OFDM transmission and we describe the emplacement of pilots signals according to the LTE standard. In the second section, we present the model of SSPA power amplifier adopted in this work. In the third section we describe the LMS and
the Volterra model of channel estimation. In section VI, we evaluate the performance of Volterra channel estimation in term of BLER, EVM(%) VS SNR and constellation diagram over Ped B model channel and compare them with performance of LMS techniques.

II. FRAME STRUCTURE

The LTE physical layer uses many technologies to maintain requirements for high data rates and spectral efficiency. The design of the physical layer and system parameters are well matched with the characteristics of mobile propagation channel to allow optional downlink and uplink frequency selective scheduling thus enhancing throughput performance. Modulation and coding according to the LTE standard maximizes throughput system and increases overall cell capacity.

In the time domain, within LTE several time intervals are expressed as multiples of a basic time unit $T_s = 1/30720000$. The frame length is 10 ms ($T_{frame} = 307200 \cdot T_s$). Each frame is divided into ten equally sized subframes of 1 ms in length ($T_{subframe} = 30720 \cdot T_s$). Each subframe is composed of two equally slots of 0.5 ms ($T_{slot} = 15360 \cdot T_s$). Each slot is consisted of a number of OFDM symbols according to the cyclic prefix type, seven with normal cyclic prefix or six in extended cyclic prefix. Figure 1 shows the frame structure for LTE Downlink system in FDD mode. The symbol time is $T_u = 2048 \times T_s = 66.7 \mu s$. For the normal cyclic prefix mode, the first symbol has a cyclic prefix of length $TCP = 160 \times T_s = 5.2 \mu s$. The remaining six symbols have a cyclic prefix of length $TCP = 144 \times T_s = 4.7 \mu s$. For the extended cyclic prefix mode, the cyclic prefix is $TCP-e = 512 \times T_s \approx 16.7 \mu s$. The CP is longer than the typical delay spread of a few microseconds typically encountered in practice as shown in Figure 2.

![Figure 1: LTE downlink structure.](image)

In the frequency domain, within LTE standard the number of sub-carriers $N$ varies between 128 to 2048, according to the LTE channel bandwidth from first bandwidth (1.5 Mhz) to the last bandwidth (20 Mhz). The sub-carrier spacing is $D_f = 1/T_u = 15 \text{ kHz}$. The sampling rate is $f_s = D_f \cdot N = 15000 \text{ N}$. LTE parameters have been chosen such that FFT lengths and sampling rates are easily obtained for all operation modes while at the same time ensuring the easy implementation of dual-mode devices with a common clock reference.
III. OFDM SYSTEM MODEL

The binary information is first grouped, coded using turbo encoder, and mapped using the complex constellation QPSK, 16 QAM or 64 QAM. After inserting the pilot symbols according to the LTE standard, an N inverse Fast Fourier transform (IFFT) block transforms the modulated data into time domain. After the IFFT block, a cyclic prefix of time length TG, chosen to be larger than the expected delay spread, is inserted to remedy to the intersymbol and intercarrier interferences. Transmission is made through a multipath channel over a multiple antenna system. Multiple antennas can be used in the transmitter and the receiver; consequently, multiple-input multiple-output (MIMO) encoders are needed to increase the spatial diversity or the channel capacity. Applying MIMO allows us to get a diversity gain to remove signal fading or getting a gain in terms of capacity. Generally, there are three types of MIMO receivers, as presented in [8]. At the receiver, after removing the CP, the FFT block transform the data back to the frequency domain. Then the reference symbols are extracted and the received symbols are estimated. Finally, the binary information data is obtained back after the demodulation and channel decoding.

IV. PILOTS SIGNAL FOR LTE DOWNLINK SYSTEM

In the LTE standards, pilots are placed on a well-defined ways to cover up the frequency and time domain. The location of the pilots for MIMO-LTE system 2*2 is shown in following figures.
It can be seen that, through the first antenna, pilots are disposed, respectively, in OFDM symbols numbers 1, 5, 8 and 12 while for the second antenna, they are placed in the same OFDM symbols, but in the different subcarriers index. The benefit of those positions is they allow a better coverage of the frequency and the time domain and eliminate the risk of interference in reception [9].

Solid State Power Amplifier: Limiter Model

In this paper we choose a Limiter model for a SSAP power amplifier. The expression of Limiter Transfer characteristics is given as follow:

\[
g(x) = \begin{cases} 
  x & |x| \leq s \\ 
  s & |x| \geq s 
\end{cases} 
\]

where \(g(x)\) is output of the power amplifier for a particular input \(x\) and \(V_s\) is the saturation level. This model does not consider AM/PM conversion [10]. The conversion characteristics of power amplifier is given by, according to the Rapp SSPA model, by:

\[
V_{out} = V_{in} / \left(1 + (V_{in}/V_s)^{2 \rho} \right)^{1/2 \rho} 
\]

where \(V_{in}\) is the complex input, \(V_{out}\) is the complex output, \(V_s\) is the saturation level and \(\rho\) is “knee factor” that controls the transition from the linear part to the saturation part of characteristic curve (a typical value of \(\rho\) is 1). As the value of knee factor increases the SSPA model approaches the Limiter Model. SSPA model is very accurate in defining the transfer characteristics of solid state amplifiers which are now mainly used in transmitters [10].

V. LEAST MEAN SQUARE (LMS) ALGORITHM

In this paper, we used the LMS algorithm in order to combat the signal distortions introduced by interferences and multipath channel: least mean square algorithm is a linear adaptive filtering belongs to the family of stochastic gradient algorithms [11, 12]. The LMS algorithm includes two parts: the first one, the output of a transversal filter is calculated as a function of tap inputs and the error term is generated based on the difference between the filter output and the data learning. In the second part, the adjustment of the tap weights is performed according to the error term. The LMS algorithm forms a feedback loop against the error term is fed back. The filter produces an output.
signal and the difference between the output and the learning data is obtained. For an adaptive linear filter with L number of coefficients, its output signal at certain time n can be represented as the follows [11][12]:

$$y(n) = \sum_{i=0}^{L-1} w_i(n)x(n)$$  \hfill (3)

$x(n)$ is the filter input signal, $w_i(n)$ is the tap coefficient at time n.

The coefficients $w_i(n)$ and the input signals $x(n)$ in are expressed in a vector form, as follow:

$$w_i(n) = [w_0(n), w_1(n), \ldots, w_{L-1}(n)]^T$$  \hfill (4)

$$U(n) = [x(n), x(n), \ldots, x(n-L+1)]^T$$  \hfill (5)

The desired output vector $D$ is given by:

$$D(n) = [d(1), d(2), \ldots, d(i)]$$  \hfill (6)

Assume we use the linear filter to model an unknown system. At time n, $e(n)$ is the difference between the desired signal $D(n)$ and the filter output $y(n)$ which is given below:

$$e(n) = d(n) - Y(n) = d(n) - w(n)^T U(n)$$  \hfill (7)

The problem to solve is to update the adaptive filter coefficients in (07) to minimize $E[e^2(n)]$. The coefficients vector can be updated by the gradient of the mean square error as shown in (8), where is the step size.

$$w(n+1) = w(n) - \varepsilon \nabla \{E[e^2(n)]\}$$  \hfill (8)

Since $e(n)$ is a function of filter coefficients, the gradient of the mean square error can be estimated as follows:

$$\nabla \{E[e^2(n)]\} = \nabla \{e^2(n)\} = 2e(n)\nabla \{e(n)\}$$  \hfill (9)

$$\begin{bmatrix}
\frac{\partial e(n)}{\partial w_0(n)} \\
\frac{\partial e(n)}{\partial w_0(n)} \\
\vdots \\
\frac{\partial e(n)}{\partial w_{L-1}(n)}
\end{bmatrix}$$

$$\nabla \{e(n)\} =$$

$$\begin{bmatrix}
\frac{\partial e(n)}{\partial w_0(n)} \\
\frac{\partial e(n)}{\partial w_0(n)} \\
\vdots \\
\frac{\partial e(n)}{\partial w_{L-1}(n)}
\end{bmatrix}$$  \hfill (10)
\[ \nabla \{ E[e^2(n)] \} = -2e(n) \begin{bmatrix} x(n) \\ x(n-1) \\ \vdots \\ x(n-L+1) \end{bmatrix} \quad (11) \]

VI. VOLTERRAMODEL

The RLS (recursive least squares) algorithm is an algorithm for determining the coefficients of an adaptive filter. In contrast to the LMS algorithm, the RLS algorithm uses information from all past input samples (and not only from the current tap-input samples) to estimate the (inverse of the) autocorrelation matrix of the input vector.

To decrease the influence of input samples from the far past, a weighting factor for the influence of each sample is used. A typical adaptive technique is shown in Figure 3. The Volterra filter of a fixed order and a fixed memory adapts to the unknown nonlinear system using one of the various adaptive algorithms. The use of adaptive techniques for Volterra kernel estimation has been well studied. Most of the previous research considers 2nd order Volterra filters and some consider the 3rd order case [13].

In a Volterra model, although the output signal is a polynomial combination of current and past input symbol, \( x(n) \), its output symbol is linearly dependent on the Volterra filter kernels. The input-output relations of a complex third order adaptive Volterra model at time \( n \) can be represented by (12), where \( h_i(n) \) are the linear kernels and \( h_{i,j,k}(n) \) the third order kernels at time \( n \).

\[ y(n) = \sum_{i=0}^{N} h_i(n)x(n-i) + \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=0}^{N} h_{i,j,k}(n)x(n-i)x(n-j)x(n-k) \quad (12) \]

In this section, we demonstrate how the RLS algorithm can be used to update the Volterra model coefficients. We use a third order Volterra filter to model an unknown system. In the Volterra model, the input vector is given in (13):

\[ U(n) = [x(n), x(n-1), \ldots, x(n-N), x^2(n), x(n-1), x(n-N)\ldots, x^2(n-N)]^T \quad (13) \]

And the desired output vector \( D(n) \) as shown as:
\[ D(n) = [d(1), d(2), \ldots, d(i)] \] (14)

The coefficients vector is:
\[ w = [h_0(n), h_1(n), \ldots, h_{n_{0,0}}(n), h_{n_{0,1}}(n), \ldots, h_{n_{N,0}}(n), h_{n_{0,N}}(n), h_{n_{N,N}}(n)]^H \] (15)

\( L \) is the total coefficient number. The difference between desired output and estimated output can be expressed by:
\[ e(i) = d(i) - Y(i) = d(i) - w(i)^H U(i) \] (16)

The objective of least square algorithm is to minimize the cost function:
\[ \psi(n) = \sum_{i=1}^{N} \lambda^{n-i} |e(i)|^2 \] (17)

where \( \lambda \) is the exponential weighting factor or forgetting factor, which can track the slow statistical variations of the channel [15]. It is a positive constant with value close to, but less than one [15, 14]. A special case is \( \lambda = 1 \) which corresponds to infinite memory [14], and can be used in a stationary environment [14]. The correlation matrix \( \varphi \) of the input signal and the cross-correlation vector \( \theta \) between the filter input and the desired output signals can be defined as following:

\[ \varphi(n) = \sum_{i=1}^{N} \lambda^{n-i} U(i)U^H(i) \] (18)

\[ \theta(n) = \sum_{i=1}^{N} \lambda^{n-i} U(i)d^*(i) \] (19)

The optimal value of \( w(n) \) which minimizes the cost function can be obtained from:
\[ \theta(n) = \varphi(n)^{-1} w(n) \] (20)

The correlation matrix in (28) and the cross-correlation vector in (21) can be reorganized as following:

\[ \varphi(n) = \lambda \sum_{i=1}^{N} \lambda^{n-i} U(i)U^H(i) + U(n)U^H(n) = \lambda \varphi(n-1) + U(n)U^H(n) \] (21)

\[ \theta(n) = \lambda \sum_{i=1}^{N} \lambda^{n-i} U(i)d^*(i) + U(n)d^*(n) = \lambda \varphi(n-1) + U(n)d^*(n) \] (22)

We can use the matrix inversion lemma methods to get the inverse matrix of \( \varphi(n) \) [24].

Assume a positive definite matrix \( A \) can be expressed as:
Where, A and B are positive definite M-by-M matrices. D is the N-by-M positive-definite matrix and C is an M-by-N matrix. The inverse matrix of A can be calculated as:

\[ A = B^{-1} + CD^{-1}C^H \]  \hspace{1cm} (23)

If we let A, B, C and D equivalent to following:

\[ A = \varphi(n) \]
\[ B = \lambda \varphi(n-1) \]
\[ C = u(n) \]
\[ D = 1 \]

Substituting in (34), we can get:

\[ \varphi^{-1}(n) = \frac{1}{\lambda \varphi(n-1)} - \frac{\varphi^{-1}(n-1)U(n)U^H(n)\varphi^{-1}(n-1)}{\lambda^2 + \lambda U^H(n)\varphi^{-1}(n-1)U(n)} \]  \hspace{1cm} (25)

Let the inverse correlation matrix \( \nu(n) = \varphi^{-1}(n) \), and then we have:

\[ \nu(n) = \frac{1}{\lambda} \nu(n-1) - \frac{1}{\lambda} k(n)U^H(n)\nu(n-1) \]  \hspace{1cm} (26)

Where \( k(n) \) is the gain vector, and

\[ k(n) = \frac{\nu(n-1)U(n)}{\lambda + U^H(n)\nu(n-1)U(n)} \]  \hspace{1cm} (27)

The procedure of updating estimate of Volterra filter coefficient vector \( \hat{w}(n) \) with the RLS algorithm can then be derived and is summarized in Table 3 [14, 15].

The figure 6 depicts the Volterra equalizer structure used in this simulation, the first step consist of the determination of Volterra Error(16) derived from reference signal. Afterwards, we estimate the totality of Volterra Error matrix using a polynomial interpolation of Lagrange [16]. Finally, the full Volterra equalizer exploits this error matrix to estimate received signal using the others relations.
VII. SIMULATION RESULTS

In order to evaluate the Volterra equalization, simulation of LTE Downlink System was executed in presence of nonlinearity effects introduced by the power amplifier of OFDM Transmitter. The simulation parameters are based on the LTE standard and are summarized in Table I. The transmission was made over Pedestrian B channel model, a 1.5Mhz bandwidth was assumed in this simulation, we choose a 16-QAM constellation according to the LTE standard (CQI=7) each SNR values includes the transmission of 1000 frames, we use also a 2*2 MIMO system. The performance of this system using Volterra equalization was compared with the LMS equalization in term of BLER vs SNR, EVM(%) VS SNR and the diagrams of constellation.

Table I parameters simulation

<table>
<thead>
<tr>
<th>Transmission Bandwidth</th>
<th>1.4 MHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of sub frames</td>
<td>1000</td>
</tr>
<tr>
<td>Number of transmit antennas</td>
<td>2</td>
</tr>
<tr>
<td>Number of receive antennas</td>
<td>2</td>
</tr>
<tr>
<td>Channel Quality indicator</td>
<td>7</td>
</tr>
<tr>
<td>Modulation order</td>
<td>16QAM</td>
</tr>
<tr>
<td>Channel type</td>
<td>ITU-Ped B</td>
</tr>
<tr>
<td>Equalization</td>
<td>Volterra&amp; LMS</td>
</tr>
</tbody>
</table>

In this subsection, we analyze the performances of LTE Downlink transmission in term of received constellation diagrams and EVM(%) vs SNR for LMS and Volterra channel estimation algorithm. For comparison purpose, the adaptive linear equalizer is also included in the simulation to evaluate the performance of nonlinear equalizers. The received signal constellation diagram for nonlinear equalization with the method of Volterra and the linear equalization with LMS technique are shown in Fig. 2 and Fig.3. Due to the nonlinearity effects introduced by power amplifier of OFDM
Transmitter and the fading distortions introduced by the multipath channel, the constellation diagram has become scattered and has phase and amplitude distortions. The linear kernels account for the interference introduced by OFDM technology and the fading distortion of multipath channel, while the third order kernels can account the nonlinear effects introduced by power amplifier [12]. Since linear equalizer has no nonlinear terms, its capability of removing the nonlinearity is restricted. As shown in the constellation diagram, it is obvious that the nonlinear equalizer of full Volterra outperforms the linear equalizer.

![Figure 8: EVM(%) for Volterra and LMS equalization.](image)

In the second simulation, we have to evaluate percentage of Error Vector Magnitude against SNR. EVM measurement can provide a great deal of insight into LTE modulation performance. EVM is expressed as the difference between the vector of an ideal symbol and the symbol under test. It’s the percentage of error that indicates how far the symbol is transmitted from its ideal position. It can be seen, over the Fig 5 that the EVM measurement performance for Volterra algorithm is clearly better than LMS algorithm. In fact for a SNR=10, the percentage of error of full Volterra equalizer is almost 5% whereas it is almost 20% for LMS equalizer.

In the third simulation, we present performance in term of the BLER vs SNR with full Volterra equalizer and LMS equalizer for LTE Downlink system and the resulting BLER curves are shown in Fig. 4

![Figure 9: BLER vs SNR for Volterra , LMS and Perfect channel estimation](image)
As indicated in Fig. 4 Full Volterra equalizer has better performance than LMS equalizer. In fact for a BLER=10^{-2} we have a great gain of 10 dB for the full Volterra equalization compared to LMS method considering the third order kernels.

**Figure 9: Throughput vs SNR for Volterra , LMS and Perfect channel estimation**

Figure 6 depicts the throughput versus SNR where the channel estimation algorithm for Volterra and LMS techniques are compared. We observe that the Volterra algorithm improves an approximately gain of 5 dB for 1 Mbit/s throughput compared to LMS channel estimation algorithm.

**VIII. CONCLUSION**

This paper presents the investigation of the nonlinearity, the interference and the fading effects in LTE Downlink System. The Volterra model based electrical equalizer has been shown capable of compensating the nonlinearity effect introduced by the power amplifier of OFDM transmitter. The biggest disadvantage of a Volterra model based nonlinear compensator is its complexity. A considerable amount of Volterra model coefficients is usually required to model a nonlinear system. Consequently, it may not be feasible to apply a Volterra model based compensator in real-time signal processing applications. One possible solution is to identify the most significant coefficients of a Volterra model and delete all of the insignificant coefficients from the Volterra model. The resulting Volterra model is referred to as a sparse Volterra model by some researchers and this will be the purpose of our future work.

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