International Journal of Current Trends in Engineering & Research (IJCTER)
e-ISSN 2455–1392 Volume 2 Issue 2, February 2016 pp. 89 - 100
Scientific Journal Impact Factor : 3.468
http://www.ijcter.com

On Fractal Dimension: A Review

Nibedita Mohanty[1], Sarojananda Mishra[2]

[1,2] *Department of Computer Sc & Engg, Indira Gandhi Institute of Technology, Sarang, Dhenkanal, Odisha (India)*

ABSTRACT—The fractal dimension (D) is a central construct developed in fractal geometry to describe the geometric complexity of natural phenomena as well as other complex forms. This paper provides a survey of several commonly used methods for estimating the fractal dimension. These methods include Walking divider Method,Box counting method, Prism counting method, Differential box counting method, Epsilon-Blanket method,Perimeter area relationship, Fractional Brownian Motion ,Power spectrum method and Hybrid methods. A method for calculating the dimension by using L-System is given. This paper discusses regarding various methods and a comparative study of different methods is presented including its merits and demerits.

KEYWORDS—Fractal Geometry, Fractal Dimension, Box counting method, L-System, Comparative Study

I. INTRODUCTION

A fractal is defined as a rough or fragmented geometric shape that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole[1]. Benoit B. Mandelbrot gives a mathematical definition of fractal as a set for which the Hausdorff Besicovich dimension strictly exceeds the topological dimension[2].A line has a dimension of 1 and a square a dimension of 2 , many curves have an "in-between" dimension related to the varying amounts of information they contain.Those in-between dimensions as the Hausdorff-Besicovitch dimension. A fractal is a self-similar pattern with infinite detail[3].Nature presents a large variety of fractal forms including trees, rocks, mountains cloud,water courses, coastlines, galaxies snowflakes[4].There are many mathematical structures that are fractals; e.g. Sierpinski triangle, Koch snowflake, Peano curve, Mandelbrot set, Julia set and Lorenz attractor Fractals are generally self-similar and independent of scale .

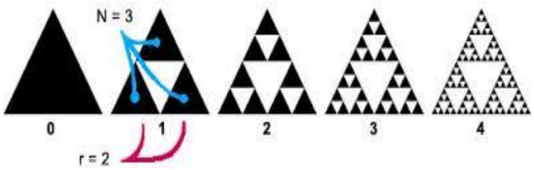


Fig.1. [Sierpinski Triangle]

An equilateral triangle is done. The mid points of the three sides are collected and the resulting inner triangle is removed. The above process is repeated for all the triangles which is shown in fig.1.

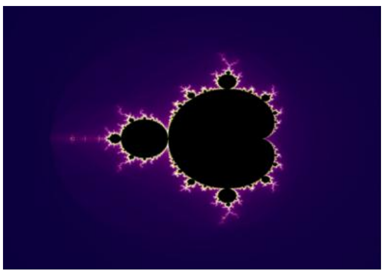


Fig.2. [Mandelbrot Set]

Mandelbrot set can be constructed by iterating the formula

$$z_{n+1}=z_n^2+c$$

Where z is a complex number $z_0=0$.For different values of c, the trajectories either: stay near the origin, or "escape" which is shown in fig.3.

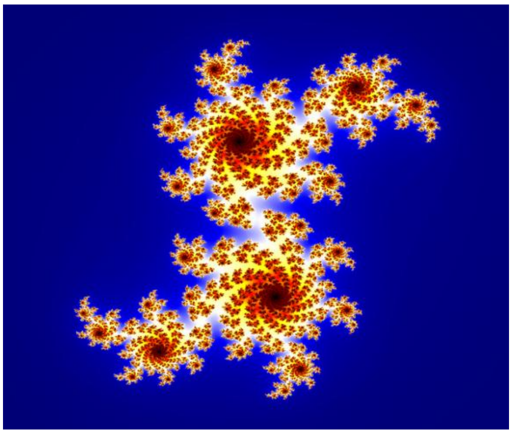


Fig.3. [Julia Set]

Julia geometrical fractal is always related to the Mandelbrot fractal. This is because they both start from the same somewhat complex function. Julia set is generated from the formula z^2+c. The constant c varies in the complex plane which is shown in fig.4.

A fractal figure is having certain properties. A set with detailed fine structure.All magnifications of some or all of a fractal set reveal intricate detail.All magnifications of the fractal reveal a set that is exactly the same or statistically or asymptotically the same. Each fractal is composed of multiple iteration of a single shape infinite number of times. An iteration is a mathematical (or geometrical) operation that is repeated a certain number of times to reach (or approximate) the final result. Fractals cannot be described easily in terms of Euclidean geometry. The most important property of a fractal is its self-similarity property and its non-integer dimension. Roughly dimension indicates how much space a set occupies near to each of its point [2].

Fractals are not just complex shapes but anything that appears random and irregular can be a fractal. A very important application is to reproduce the natural image because many natural things, such as plants are very complex and exhibit some self-similarity. The most useful use of fractals in computer science is the fractal image compression. The images are compressed much more than by usual ways for e.g.: JPEG or GIF file formats. When the picture is enlarged, there is no pixelisation. Human body is full of fractals, fractal math is used to quantify, describe diagnose and perhaps help to cure the diseases. Everything exists on this world is a fractal.

II. FRACTAL GEOMETRY

Fractals in nature are so complex and irregular that it is difficult to model them by using classical geometry. Fractals are geometric objects, exactly like the circle or triangle, they have some different properties. A substantial difference between a euclidian and fractal geometries object is the way it builds.The euclidian plane with a curve,the fractal however is not based an equation but on an algorithm.This means that there is a method not necessarily numeric,which must be used to draw the curve.In addition,the algorithm is never used once.The procedure is iterated a number of times infinity:each iteration, the curve gets closer and closer to the end result and after a certain number of iterations the human eye is no longer able to distinguish the changes, so when you actually draw a fractal, you can stop after a reasonable number of iterations.A curve is called fractal if enlarging any part of the curve displays a particular set of equally rich and complex than the previous, this process of “zoom” can be continued indefinitely.

III. FRACTAL DIMENSION

Fractal dimension is a mathematical concept which belongs to fractional dimensions. A fractal dimension is a ratio which provides how detail in a pattern changes with the scale at which it is measured[14].It gives an objective means for quantifying the fractal property of an object and comparing objects observed in the natural world.With the help of fractal dimension it can be defined how irregular a fractal curve is. Let us take a very irregular curve however which wanders to and from on a surface (e.g. a sheet of paper) or in the three dimension space. Several curves like this: the roots of plants, the branches of trees, the branching network of blood vessels in the human Body, the lymphatic system, a network of roads etc.Thus, irregularity can also be considered as the extension of the concept of dimension. The dimension of a curve is a number that characterises how the distance grows between two given points of the curve while increasing resolution. That is, while the topological dimension of lines and surfaces is always 1 or 2, fractal dimension can also be in between. A number of methods are available to calculate fractal dimension and new methods are always being derived. These methods includes Walking-Divider Method,Box Counting Method, Differential Box Counting Method,Prism-Counting Method,Epsilon Blanket Method, Fractional Brownian Motion, Perimeter Area Relationship, Robust fractal estimator, Fourier power spectrum method, Isarithm method, Variogram Method,Hybrid Method.

But most of the methods have their practical and theoretical limitations.A problem which affects many of the methods is that the dimension of the whole may not be equal to the dimension of the part.Choose a method which can be easier to implement and require less computational power.

3.1. Walking-Divider method

This method is developed by shelberg in 1982.It requires a chord length (step) and calculates the number of chord length required.In each step vary the step size (N) and count the number of chord lengths(S) required to cover a fractal curve.By plotting the graph against step and length and using least squre estimator fractal dimension (D) can be calculated.

$D = \log N / \log S$………….. (1)

This algorithm is time consuming because at each step it calculates the number of chord length.

3.2. Box counting method

This method is developed by Voss in 1985.This method can be applied to any dimensional set.This method is the widely applicable method because it is easy to implement and quite flexible and robust. Partition the fractal image with a d-dimensional fixed grid of squred sized boxes of equal size r. The number N(r) of nonempty boxes of size r needed to cover the fractal structure depends on r

$N(r) \approx r^{-D}$…………………… (2)

Where D is nothing but the box counting dimension.The box counting algorithm hence counts the number N(r) for different values of r and

Plot the log of the number N(r) versus the log of the actual box size r. The value of the box-counting dimension *D* is estimated from the formula,

$$-D=\lim_{r}\frac{\log N(r)}{\log r} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

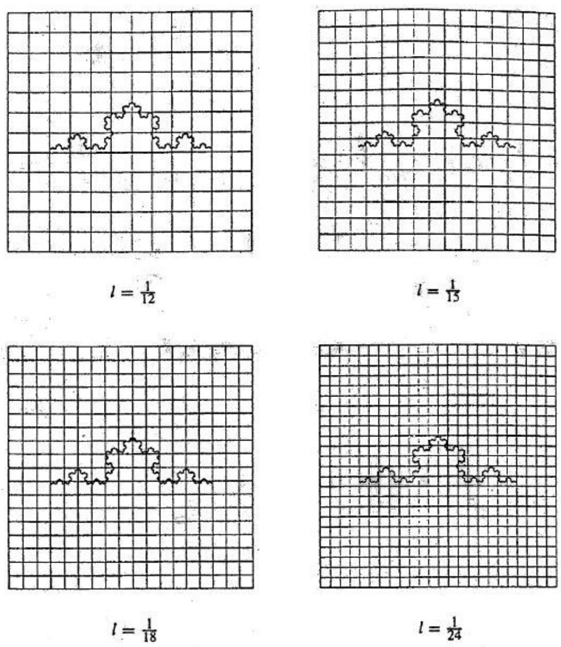
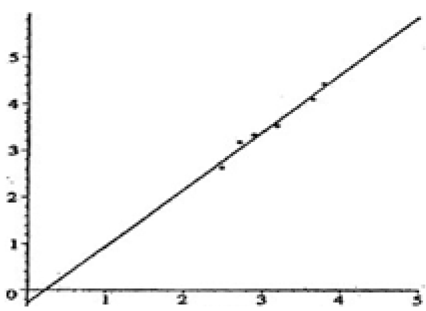


Fig. 4. [Example of Koch curve and dividing it into squred sized box]

Calculation of log r and logN(r):

R	12^{-1}	15^{-1}	18^{-1}	24^{-1}	38^{-1}	44^{-1}
N(r)	14	24	28	34	60	83
log r	2.489	2.7081	2.8904	3.1781	3.6376	3.7842
logN(r)	2.6391	3.1781	3.3322	3.5264	4.0943	4.4188

Plotting the graph against logN(r) in x-axis and log r in y-axis



Hence slope of the line of best fit of N(r) versus r yields the dimension. Here slope ≈ 1. 2246, thus D ≈1.2246

A major limitations of box counting method lies on the fact that the counting process of nonempty boxes implies its use only for binary images rather than grey scale ones.An extension of the standard approach to gray scale images is called the Differential Box Counting method(DBC).It has been proposed by N.Sarkar and Chaudhuri in 1994.

3.2.1. Differential-Box-Counting Method (DBC)

Consider a gray level image M×M as a 3-D spatial surface where(x,y) denotes the pixels spatial coordinates and the third axis z the pixels gray level.The M×M image is divided into s×s sized boxes where s varies from[M/2….1].Then scale of each block is r=s. On each block there is a column of boxes of size s×s×s'.where s' denotes the height of a single box.Let G be the total number of grey levels in I, hence s' is defined by the relation G/s'=M/s [7].Assign the number 1, 2, 3… to the boxes to group the grey levels. Let the maximum
and minimum grey level of the image in the (i, j) th grid fall in box number k and l, respectively.

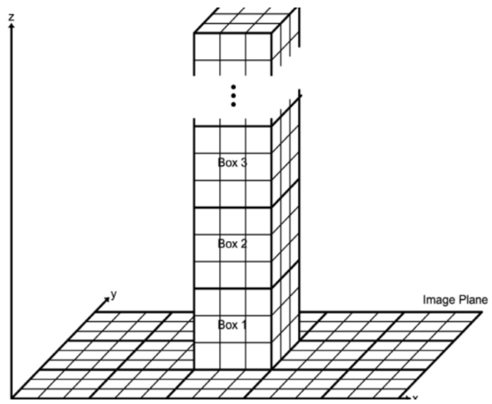


Fig. 5. [Example of DBC method application for determining the number of boxes of size s × s × s, when s = 3.]

The number of boxes covering this block is calculated as

n_r(i, j)=l-k+1………… (4)

For all the blocks

$N_r = \sum_{i,j} n_r$(I, j)………. (5)

The value of n_r is computed for different box size s.Then plot the graph by taking the values of N_r versus r in a log plot.

3.3. Prism Counting Method

This technique is similar to box counting technique.This method calculates the area based on four triangles defined by the corner points followed by summation over a grey level surface. The triangles define a prism based on the elevated corners and a central point computed in terms of the average of the four corners. A bilogarithmic plot of the sum of the prisms areas for a given base area gives a fit to a line whose slope is β in which

D = 2- β ……………….. (9)

Here D is the fractal dimension.

The limitations of this method are it is slower due to the number of multiplications implied by- the calculation of the areas.

3.4 Epsilon-Blanket method

This method is developed by Pele in 1984.In this method the fractal dimension of Curves/surfaces is computed using the area/volume measured at different scales. In the case of curves, the set of points whose distance from a curve which is not more than a small scale, ε is considered. This gives a strip of width 2ε that surrounds the curve. The length of the curve L (ε) is calculated from the strip area a (ε) by

$$L(\varepsilon) = A(\varepsilon)/2\varepsilon \ldots\ldots\ldots\ldots (10)$$

The fractal dimension, *D* is computed using the relation

$$L(\varepsilon) \propto \varepsilon^{(1-D)} \ldots\ldots\ldots\ldots (11)$$

In the case of surfaces, the set of points in three dimensional space which is not more than ε from the surface, gives a 'blanket' of volume V(ε) whose width is again 2ε. The surface area is given by

$$A(\varepsilon) = V(\varepsilon) / 2\varepsilon \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

And D can be computed as

$$A(\varepsilon) = \varepsilon (2''D) \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

3.5 Fractional Brownian motion

In probability theory fractional Brownian motion (fBm), also called a fractal Brownian motion is a generalization of Brownian motion.Unlike classical Brownian motion, the increments of fBm need not be independent. fB_m is a continuous-time Gaussian process $B_H(t)$ on [0, T], which starts at zero, has expectation zero for all t in [0, *T*], and has the following covariance function.

$$E[B_H(t)\, B_H(s)] = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t - s^{2H}|) \ldots (14)$$

Where H is a real number in (0, 1), called the Hurst index or Hurst parameter associated with the fractional Brownian motion. The parameter 0<H<1, defines the scaling behaviour. The value of H determines what kind of process the fB_m is.

- If H = 1/2 then the process is in fact a Brownian motion or Weiner process.
- If H > 1/2 then the increments of the process are positively correlated.
- If H < 1/2 then the increments of the process are negatively correlated.The increment process.

$$X(t) = B_H(t+1) - B_H(t) \ldots\ldots (15)$$

Is known as fractional Gaussian noise.

There is also a generalization of fractional Brownian motion: n-th order fractional Brownian motion, abbreviated as $n\text{-}fB_m$. $n\text{-}fB_m$ is a Gaussian, self-similar, non-stationary process whose increments of order n are stationary. For n = 1, $n\text{-}fB_m$ is classical fB_m.The main difference between fractional Brownian motion and regular Brownian motion is that the increments in Brownian motion are independent, the opposite is true for fractional Brownian motion. This dependence means that if there is an increasing pattern in the previous steps, then it is likely that the current step will be increasing as well (If H > 1/2).

3.6. Perimeter-Area Relationship

Mandelbrot (1977, 1982) introduced the concept of fractal, a geometric form that exhibits structure at all spatial scales, and proposed a perimeter-area method to calculate the fractal dimension of natural planar shapes.The perimeter-area method quantifies the degree of complexity of the planar shapes. The degree of complexity of a polygon is characterized by the fractal dimension (D), such that the perimeter (P) of a patch is related to the area (A) of the same patch by

$$P=\sqrt{A}^{D} \quad \text{(i.e. } \log P \approx \tfrac{1}{2} D \log A\text{)} \ldots. (16)$$

For simple Euclidean shapes (e.g., circles and rectangles), $P \approx \sqrt{A}$ and $D = 1$ (the dimension of a line).

3.7. Ruler dimension estimation method

This method computes the fractal dimension of a line as a function of two measurements taken while "walking" the fractal line in a number of discrete steps. We take as unity the distance between the beginning and the end of the fractal line to be walked. The first measurement is p_1, the length of the step used, or pitch length, which must be constant during the whole walk. The second is the number of steps needed to reach the end of the walk by following the fractal curve, C

We call D_{p_1} the number for which the following relation holds:

$$N(p_1) \approx p_1^{D_{p_1}} \quad \ldots. (17)$$

If we take logarithms on both sides of this equation, we obtain

$$\log N(p_1) = - D_{p_1} \log(p_1) \quad \ldots. (18)$$

The fractal dimension is the limit of D_{p_1} when p_1 tends to zero:

$$D_{p_1} = \lim_{p_1} \frac{\log N(p_1)}{\log p_1} \quad \ldots. (18)$$

3.8. The robust fractal estimator

The robust fractal estimator was proposed by Clarke and Schweizer (1991) in an attempt to provide stability in the computation of D. Using the walking-dividers method, the robust fractal estimator computes for each cell the D of each profile in both the east–west and north–south directions and places the average of the two in a new array. The D of the entire surface is obtained by combining the D values of each cell using a weighted average and adding one (Clarke and Schweizer 1991). Clarke and Schweizer (1991) noted that the robust fractal estimator is primarily designed to calculate D for natural surfaces using data from USGS DEMs, but it should work equally well on any gridded surface data. Applications of this estimator to image analysis seem limited, however. As such, little is known about its performance. In their paper, Clarke and Schweizer reported that for the same datasets used in their study, the robust fractal estimator appeared to consistently yield a lower D value than those obtained from the triangular prism and variogram methods. In a discussion of how to select the largest step size when using the divider method on self-affine curves, Klinkenberg (1994) pointed out that the low estimated D values (close to one) reported in Clarke and Schweizer (1991) may be due to the fact that they used steps sizes that spanned the crossover length. Note that the crossover length refers to the range of scale within which the computed D is representative of the local fractal dimension of a self-affine feature.

3.9. The Fourier power spectrum method

Another technique for computing the D of surface features is the use of Fourier analysis. The Fourier method uses the power spectrum derived from the surface by Pentland 1984, Burrough 1981. It can be shown that the Fourier power spectrum *P (f)* of a fractional Brownian function (f) is proportional to f^{-2h-1}.The fractal dimension of the profile $D_{Profile}$ is obtained from the slope of the regression line of the log-log plot of P*(f)* versus f.
The D of the surface is computed as

$$D = D_{Profile} + 1 \quad \ldots. (19)$$

3.10. The isarithm method

The isarithm method (Shelberg et al. 1983) is based on the premise that the complexity of isarithm or contour lines may be used to approximate the complexity of a surface[6]. Briefly, the method works in the following way. Starting with a matrix of z-elevations, an isarithm interval is selected and isarithm lines are constructed on the surface. For each isarithm line, its lengths are calculated in terms of the number of boundary cells over a number of step sizes, log (number of boundary cells) is regressed against log (step sizes), and the slope of the regression line is used to derive the D of the isarithm line. This process is repeated for every isarithm line. The surface's D is obtained by averaging the D values of all the isarithm lines that have $R^2 \geq 0.9$ and adding 1. To implement the isarithm method, a data matrix of a given number of rows and columns must be specified, with the following parameter input by the user: (1) the number of step sizes, (2) the isarithm interval, and (3) the direction in which the computation is implemented (row, column or both). It is possible that for a given step size, there are no boundary cells. In this case, the isarithm line is excluded from the analysis to avoid regression using fewer points than the given number of steps.

3.11. Variogram Method

The variogram method is a widely used technique for computing D of surfaces. In this method, the mean of the squared elevation (or DN) difference i.e. variance is calculated for different distances, and D is estimated from the slope (b) of the regression between the logarithms of variance and distance .

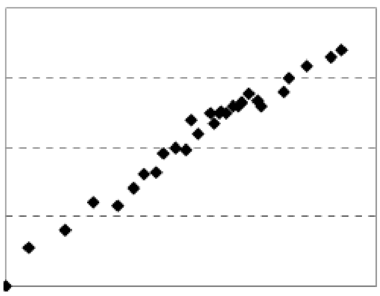


Figure 6. The log (variance in x-axis) versus log (distance in y-axis) plot used in the variogram method.

3.12. L-System

L systems are a mathematical formalism proposed by the biologist Aristid Lindenmayer in 1968 [s], are also called parallel-derivation grammars, In which derivation is not sequential but parallel. A single rule is applied at every step and as many rules as possible are applied. In L-systems the initiator maps to the axiom of the L system, and the generator becomes the set of production rules, while recursive applications of the generator to the initiator correspond to successive derivations of the axiom.

The formal definition of an L-system grammar, very similar to the definition of the Chomsky grammar, can be made by a sorted quadruple {V, T, P, S}. Where V represents the set of variable symbols, T represents the set of terminal symbols; usually the symbols {+,-, [,]} are Used, T can be an empty set, P represents the set of production rules, where P belongs to V×V*, S represents the axiom, were S belongs to V+. For calculating fractal dimension two things are required. The first is the length *N* of the visible walk that follows the fractal generator (equal in principle to the number of draw symbols in the generator string. The second is the distance d in a straight line from the start to the endpoint of the walk, measured in turtle step units (this number can also be deduced from the string). The fractal dimension is then

$$D = \frac{\log N}{\log d} \qquad \text{.... (20)}$$

Algorithm

- First the n^{th} derivation is obtained from axiom according to the production rule.
- Then the graphic interpretation of the resulting string is calculated in order to get the point reached.
- The euclidian distance between two points is calculated. The effective length (without overlapping segments) depicted is calculated.
- Finally the fractal dimension is calculated
  By dividing the effective length and distance and taking the log.

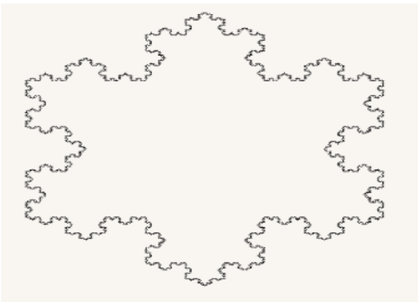


FIG 7. Von Koch snowflake curve

Eg: F=F+F—F+F
with axiom F--F-- and a turtle graphic interpretation, where {*F*} is a draw symbol and the step angle is 60 degree, represents the fractal whose fifth derivation appears in Figure 1 (Von Koch snowflake curve). The only string to be considered is F+F-+F+F .... (21).
This string describes the fractal generator. The number of steps along the walk (*N*) is the number of draw symbols in the string, 4 in this case. The distance *d* between the extreme points of the generator, computable from the string by applying to it the turtle interpretation, is 3.Therefore, the dimension is

$$D = \frac{\log 4}{\log 3} = 1.2618595071429 \ldots,$$

By using the algorithm same dimension can be calculated.

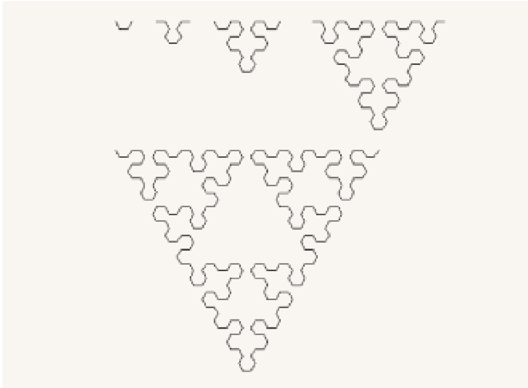


FIG 8. Fractal curve represented by two rules with the same derived dimension.

It represents the fractal whose first five derivations appear in fig 9.In this example, there are two strings to be considered:

-G+F+G-

+F-G-F+

Applying the algorithm to each of them, we obtain the same estimation of the fractal dimension 1.58496..

3.13. Hybrid Method

This method is developed by Turner in 1998. Hybrid methods calculate the fractal dimension of 2-D surfaces using 1-D methods. This approach is based on the relationship that exists between the fractal dimensions of a surface's contours (1-D fractal curves) and the fractal dimension of the surface itself.

$$D2 = 1+D1 \ldots\ldots (22)$$

Where D1 is the average of the fractal dimensions of each contour line and D2 is the fractal dimension of the surface.

IV. COMPARATIVE STUDY

Methods	Application	Demerit
Walking Devidor	Practical to length	The computation of the initial value and the procedure required to count the number of steps is time consuming
Box counting	It can be applied to any dimensional set. Mainly used to determine the area of irregular figures.	Its use only for binary images rather than grey scale images. Scale dependent
Differential-Box-Counting Method(DBC)	It calculates the dimension of grey scale images	To select an appropriate box height is quite hard. The box number calculation may lead to overestimate the number of boxes needed to cover the surface.

Prism counting	Calculates the dimension of one dimensional Signals	It is slower due to the number of multiplications implied by the calculation of the areas
Epsilon-Blanket	Estimate the dimension of curve/surfaces	Widely not applicable because it can't calculate the dimension of irregular figures
Fractional Brownian Motion	Similar to box counting	This method is dependent means if there is an increasing pattern in the previous steps, and then it is likely that the current step will be increasing as well. (If H > 1/2.)
Perimeter-Area Relationship	To classify different types Images	To calculate perimeter as well as area makes this method time consuming
Power spectrum	It is generalizable, potentially more accurate.	It only calculates the dimension of Digital fractal signals.
The Fourier power spectrum method	It provides stability in the computation of D	Applications of this estimator to image analysis seem limited
The isarithm method	Complexity of isarithm or contour lines may be used to approximate the complexity of a surface	Computed D will vary depending on whether it is measured along rows, columns, or in a non-cardinal direction. The maximum step size used may also affect the reliability of estimation results.
Variogram Method	It can be applied to both regular and Irregular data. Compared to isarithm method this is generally more reliable	Variogram often do not behave linearly at all scales, suggesting that natural phenomena are not truly fractal In variogram analysis is the Sampling strategy used to determine the point pairs.
Hybrid Method	calculate the fractal dimension of 2D images using 1D methods	Techniques must be choosen and implemented carefully
L-System	Performing operations on strings is an easier Method of computing	Fractal represented by L-systems associated with a a vector graphics interpretation converted to equivalent L-systems with a turtle graphic interpretation.

fractal dimension than computation of a limit.

V. CONCLUSION

A box-counting-based method gives best result to estimate the FD of an image. To improve the estimate accuracy, it is required to use the smallest number of boxes to completely cover the image intensity surface at each specific box dimension.

REFERENCES

[1] B.B. Mandelbert, *The Fractal Geometry of Nature* ,W. Freeman and Company CLARKE, K.C.,
[2] J.Mishra and S.Mishra, L-System Fractals, MATHEMATICS IN SCIENCE AND ENGINEERING Volume 209 Series,Elsevier, 2007.
[3] N. Sarker, B.B. Chaudhuri, An efficient differential box-counting approach to compute fractal dimension of image, IEEE Transactions on Systems, Man, and Cybernetics 24 (1994) 115–120.
[4] CLARKE, K.C., Computation of the fractal dimension of topographic surfaces using the triangular prism surface area method. Computers and Geosciences, 12, pp. 713–722,1986.
[5] Dapontz, J.S. and FOX, M.D., 1989, Fractal feature analysis and classification in medical imaging. IEEE Transactions on Medical Imaging, 6, pp. 133–142,2007.
[6] Plover, M. and Nappi, M., 'Speed-Up In Image Coding : Comparison of Methods', IEEE Trans, on image processing 9, 2000.
[7] DENNIS, T.J. and DESSIPRIS, N.G., Fractal modelling in image texture analysis. IEEE Proceedings, 136, pp. 227–235, 1989.
[8] KELLER, J.M., CROWNOVER, R.M. and CHEN, R.Y., Characteristics of natural scenes related to the fractal dimension. IEEE Transactions on Pattern Analysis and Machine Intelligence, 9, pp. 621–627,1987.
[9] SUN, W., Three new implementations of the triangular prism method for computing the fractal dimension of remote sensing images. Photogrammetric Engineering and Remote Sensing, 72, pp. 373–382,2005.
[10] PENTLAND, A. P. Fractal-based description of nature scenes.*IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 6, no. 6, p. 315-326,1984.
[11] LI, J., SUN, C., DU, Q. A new box-counting method for estimation of image fractal dimension. In *Proceedings of IEEE International Conference on Image Processing*. 3029-3022, 2006.
[12] BISOI, A. K., MISHRA, J. On calculation of fractal dimension of images. *Pattern Recognition Letters*, 2001, vol. 22, p. 631-637.
[13] Mukherjee.J, Kumar.P, Ghosh,S.K., 'A Graph- Theoretic Approach for Studying the Convergence of Fractal Encoding Algorithm", IEEE Trans, on Image Processing,19 : 357 - 365, 2000.
[14] Petgen, H., Jurgens, H., Saupe, D., Fractals for the classroom, Part one, Introduction to Fractals and Chaos, National Council of teachers of mathematics, Springer-Verlag, 1991.
[15] B.B. Chaudhuri, N Sarker, Texture segmentation using fractal dimension, IEEE Transactions on Pattern Analysis and Machine Intelligence 17,72–77,1995.
[16] Ruhl,M., Hartenstein, H. and Saupe, D., Adaptive partitioning For Fractal Image Compression',IEEE International Conference on Image Processing(ICIP'97),Santa Barbara,1997.

Ms.Nibedita Mohanty is currently pursuing her MTech degree at Indira Gandhi Institute of Technology,Sarang, Dhenkanal,Odisha. She is doing her MTech disseratation under the able guidance of Prof. Sarojananda Mishra. Her research area is characteristic study of fractal images and its applications.

Dr. Sarojananda Mishra is currently working as Professor & Head of the department of Computer Science, Engg. & Applications at Indira Gandhi Institute of Technology (IGIT), Sarang, Dhenkanal, Odisha, India. He has published more than 75 papers in International Journals and National Journals of repute. His research area focuses on fractal graphics, fractal geometry and internet data analysis. He has more than 25 years of teaching and research experiences. Many students obtained PhD degree and are continuing their PhD and MTech research work under his guidance.